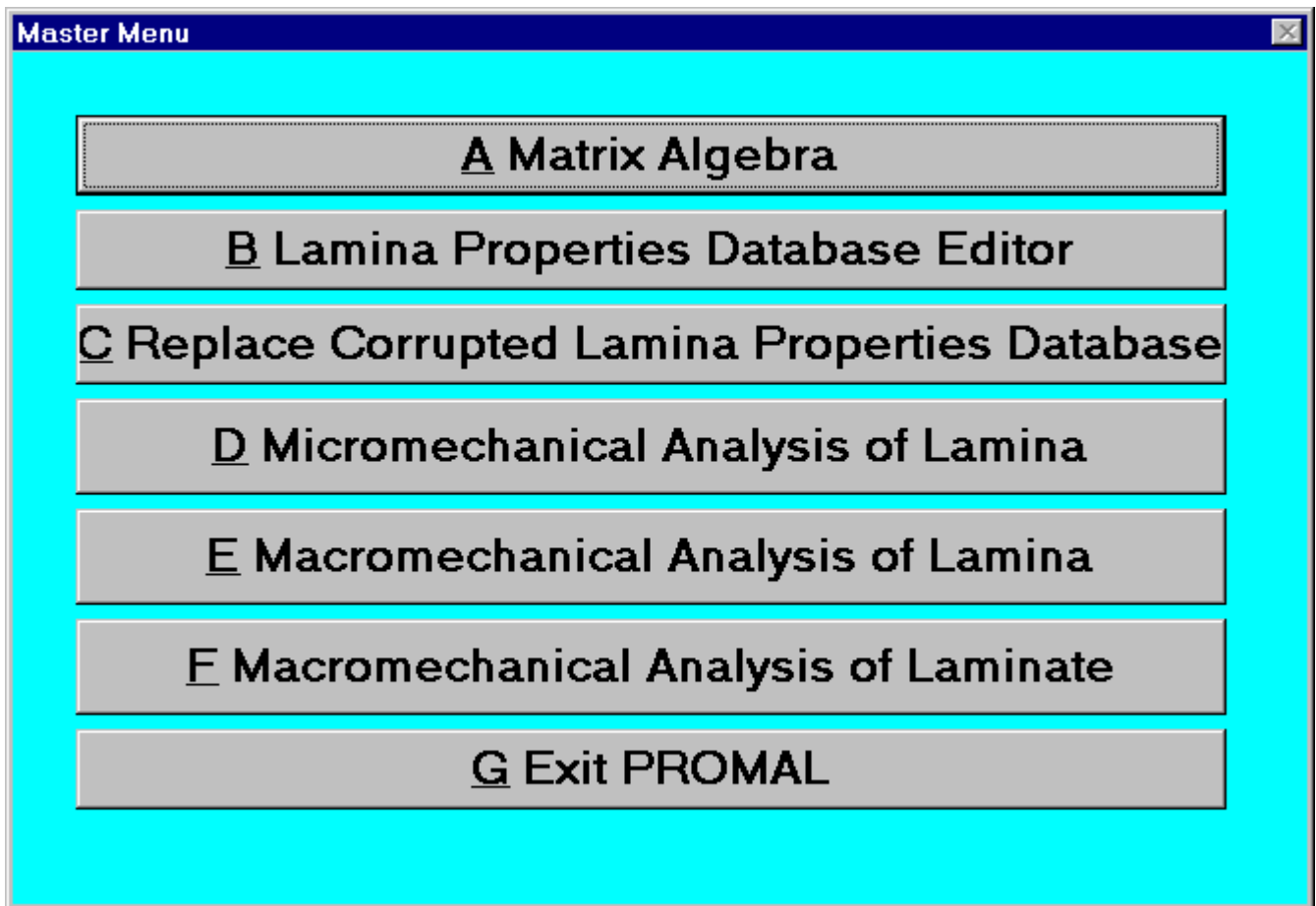# Mechanics of Composite Materials, Second Edition

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What programs are in PROMAL?

Master Menu

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The master menu screen with five separate applications from which to choose from.

Master Menu

A Matrix Algebra

B Lamina Properties Database Editor

C Replace Corrupted Lamina Properties Database

D Micromechanical Analysis of Lamina

E Macromechanical Analysis of Lamina

F Macromechanical Analysis of Laminate

G Exit PROMAL

Promal for Windows-Matrix Algebra

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Matrix Algebra

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The first program conducts multiplication of matrices, solves a set of equations and finds the inverse of a matrix. This program allows you to concentrate on the fundamentals of the course while doing homework problems, rather than spending time conducting lengthy matrix manipulations.

A Create A Matrix
B Erase All Input Matrices
C Edit A Matrix
D Solve A Set Of Equations
E Inverse Of A Matrix
F Multiplying Two Matrices
G Return To Master Menu
H Exit PROMAL

Lamina Properties Database

In the second program you can enter the material properties of a composite lamina, update or correct the properties of an existing lamina, and simply review the properties. *You can enter properties for up to a maximum of 100 different lamina materials.* Properties of three typical composite systems - Graphite/Epoxy, Boron/Epoxy, Glass/Epoxy, and two typical isotropic metals- aluminum and steel are already entered for you in the database.

Lamina Properties Database

SI System of Units	Name of Material		Graphite/Epoxy
	Longitudinal Young's Modulus	GPa	1.8100E+02
USCS System of Units	Transverse Young's Modulus	GPa	1.0300E+01
	Major Poisson's ratio		2.8000E-01
Material Name	In-Plane Shear Modulus	GPa	7.1700E+00
Graphite/Epoxy	Ply Thickness	mm	1.2500E-01
Boron/Epoxy	Longitudinal Tensile Strength	MPa	1.5000E+03
Glass/Epoxy	Longitudinal Compressive Strength	MPa	1.5000E+03
Aluminum	Transverse Tensile Strength	MPa	4.0000E+01
Steel	Transverse Compressive Strength	MPa	2.4600E+02
	In-Plane Shear Strength	MPa	6.8000E+01
	Coefficient of Thermal Expansion Dir. 1	µm/m/°C	2.0000E-02
	Coefficient of Thermal Expansion Dir. 2	µm/m/°C	2.2500E+01
	Coefficient of Moisture Expansion Dir. 1	m/m/kg/kg	0.0000E+00
	Coefficient of Moisture Expansion Dir. 2	m/m/kg/kg	6.0000E-01

New Material   **Save**   **Delete**   **Print**   **Dump**   **Return to Previous Menu**

This program enters fourteen properties- the four elastic moduli, lamina thickness, five strength parameters, two thermal expansion coefficients and two moisture expansion coefficients of a specially orthotropic lamina.

Replace Corrupted Lamina Material Database

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Sometimes, the material database for the lamina may get corrupted due to power shutdown, computer lockup, accidental rebooting, or other reasons. This option allows you to replace the material database by either:

● The database which you had the last time you exited out of the program.

● The database which you received with the disk. Note that the database on the disk has only five reserved materials.

Replacing Corrupted Lamina Properties Database

Replace Corrupted Database with Database Which Came With The Setup Disk

Replace Corrupted Database with Previously Used Database

Return to Master Menu
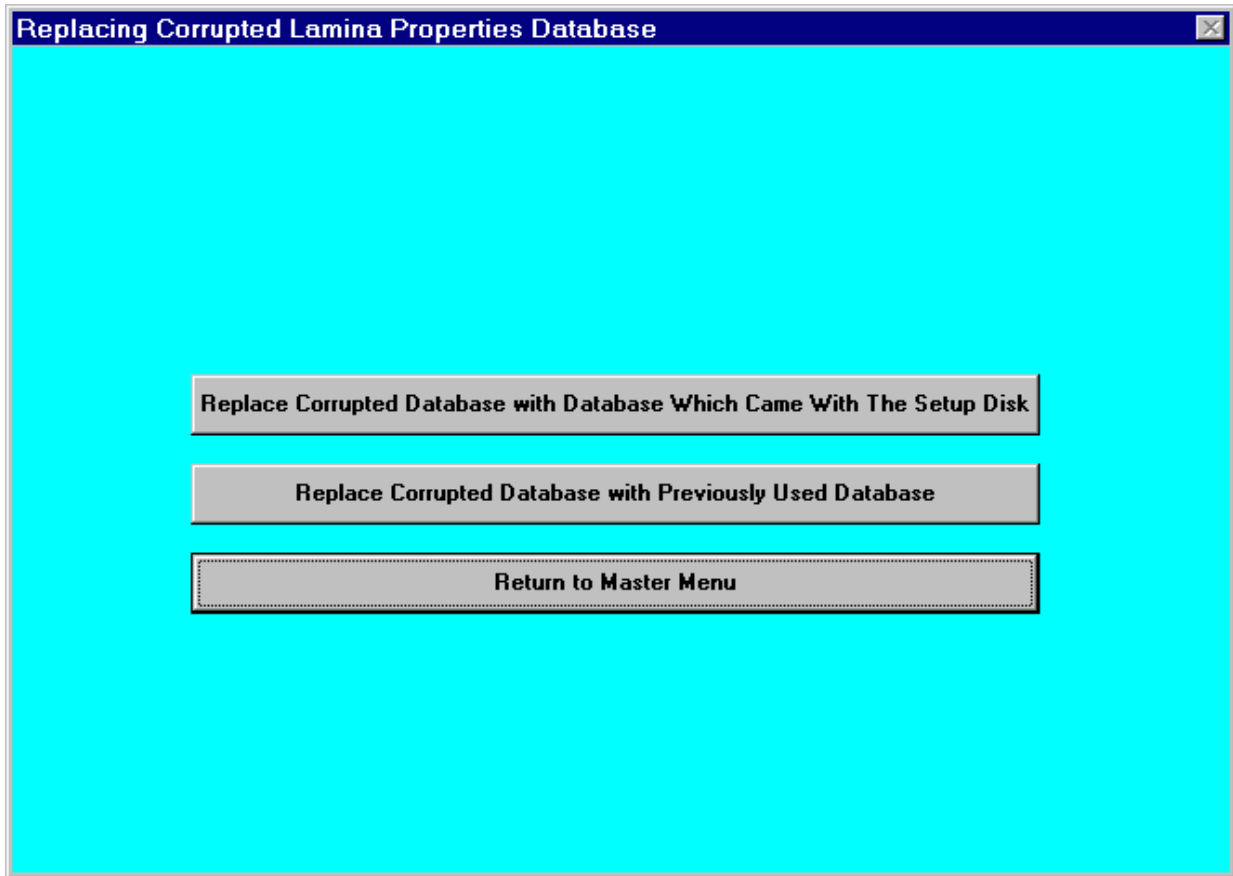
Macromechanical Analysis of a Lamina

The fourth program conducts the macromechanical analysis of a lamina. The input mechanical properties of a lamina are chosen from the lamina properties database. You can then calculate the stiffness and the compliance matrices, the engineering constants and the strength ratios as a function of the ply angle in tabular or graphical form.
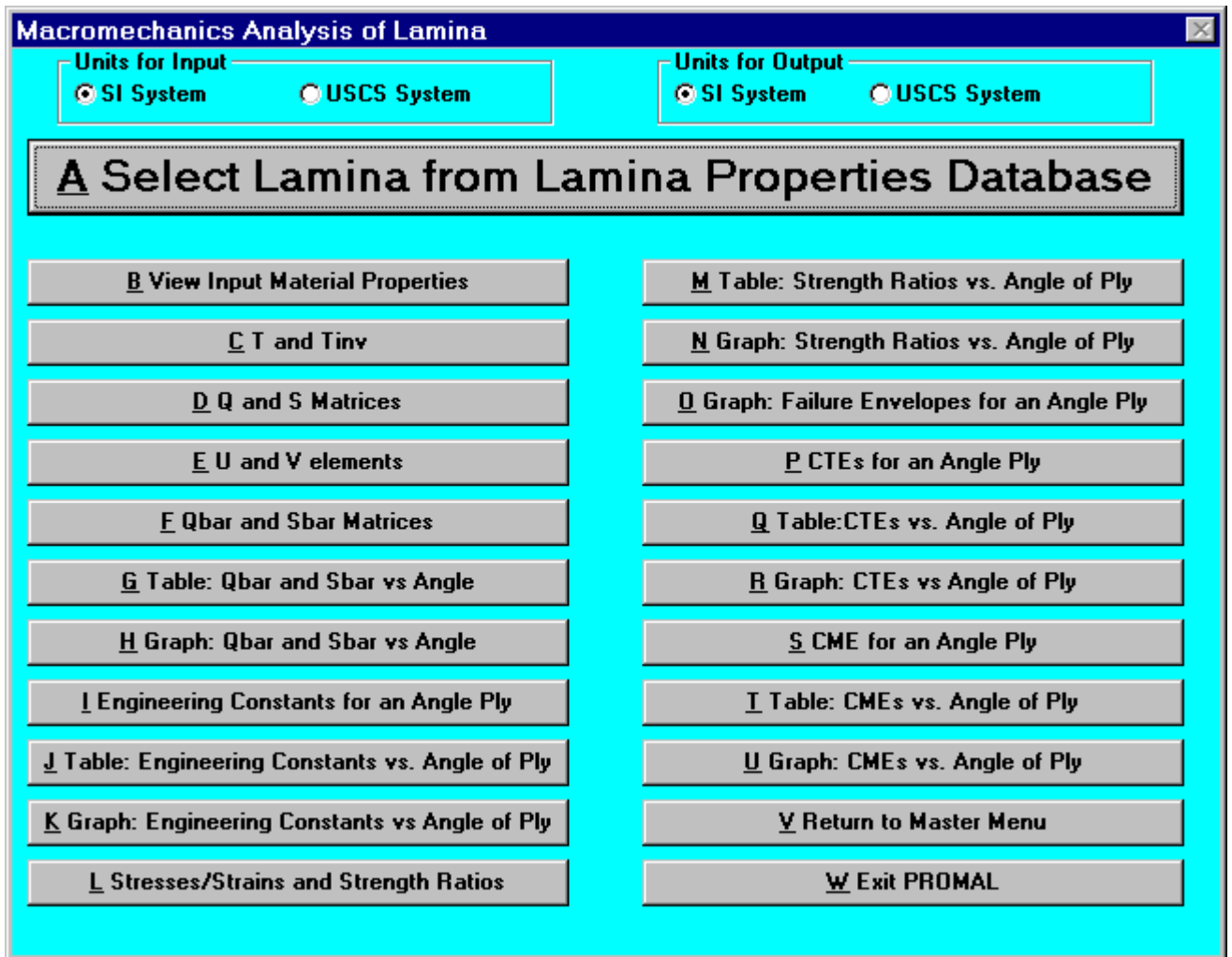
A Quick Example

Task:
Consider a typical Graphite/Epoxy lamina, find the optimum fiber angle for a load
with the following stress components. Use the maximum stress failure criterion for
your analysis. Given the following:

● LongitudinalStress = 4 KPsi
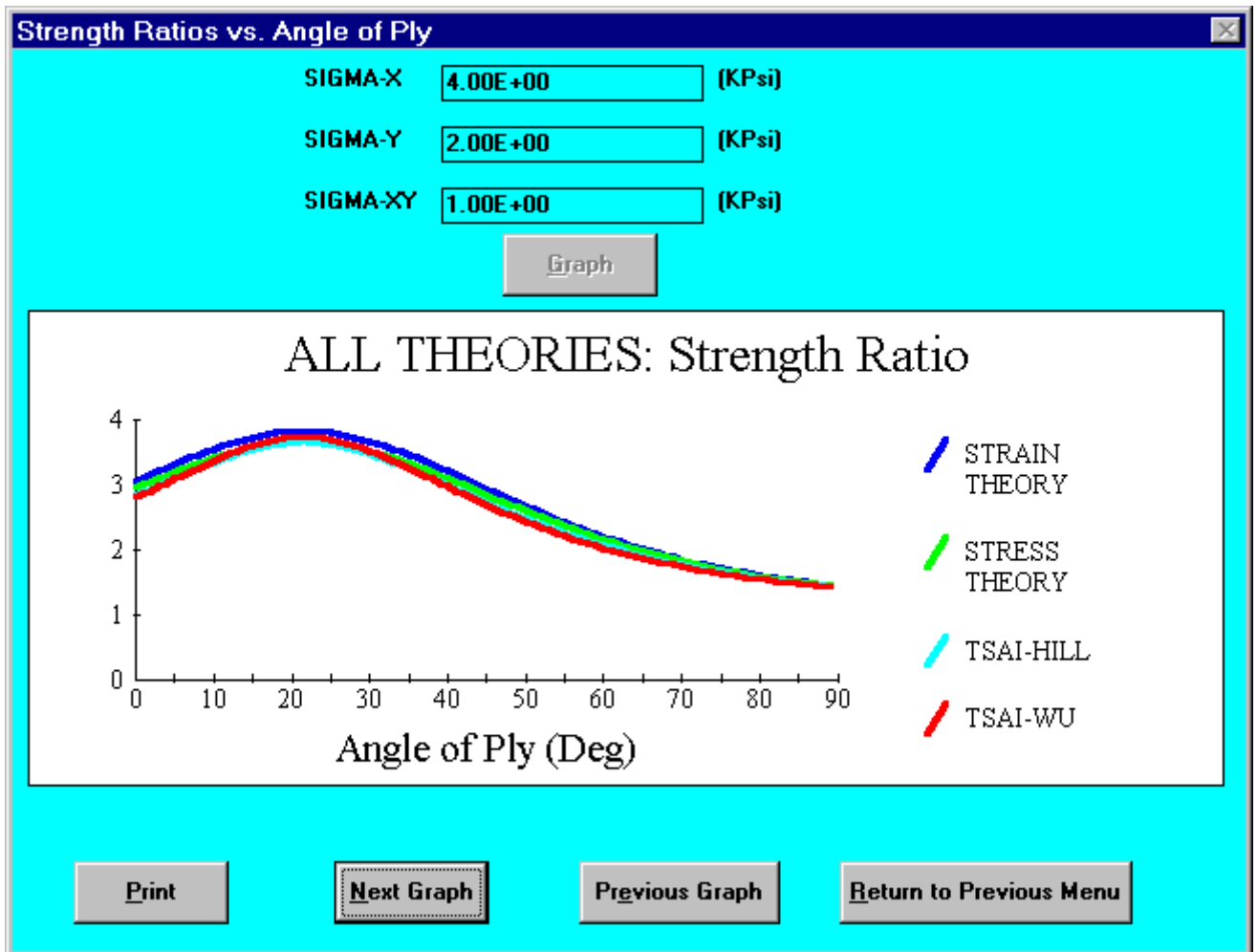● Transverse Stress = 2 KPsi
● Shear Stress = 1 Kpsi

Procedure:
Click on the Macromechanics of a Lamina Button in the Master Menu.
Click on the proper system of units for the input and output.
Click on Select Lamina from Lamina Properties Database Button to select the
material properties of the Graphite/Epoxy.
To analyze the lamina for the optimum fiber angle for the given load, select the
Graph: Strength Ratios vs. Angle of Ply Button. Enter the components of the stress

vector. Once you enter these values, you are presented with the first of four graphs showing the strength ratio as a function of the angle of ply. The second graph shows the results of using the Maximum Stress failure theory.

After inputting the given data, Promal produces the following graphical output

Strength Ratios vs. Angle of Ply

SIGMA-X	4.00E+00	(KPsi)
SIGMA-Y	2.00E+00	(KPsi)
SIGMA-XY	1.00E+00	(KPsi)

Graph

ALL THEORIES: Strength Ratio

STRAIN THEORY

STRESS THEORY

TSAI-HILL

TSAI-WU

Angle of Ply (Deg)

Print   Next Graph   Previous Graph   Return to Previous Menu

From the graph, the lamina will not fail (SR>1) for fiber angles between 10 and 30 . Further investigation using Stresses/Strains and Strength Ratios Button shows that 22.5 is the optimum fiber angle as it has a strength ratio of 3.784. So one can apply 3.784 times the stresses given above before the lamina fails.

Micromechanical Analysis of a Lamina

The fifth program conducts the micromechanical analysis of a lamina using formulas from the strength of materials approach. You enter the elastic moduli, coefficients of thermal and moisture expansion of the fiber and the matrix from the keyboard. You can then calculate the elastic moduli, thermal and moisture expansion coefficients of a unidirectional lamina for a particular fiber volume fraction. The program also displays these properties in a tabular/graphical form as a function of the fiber volume fraction.
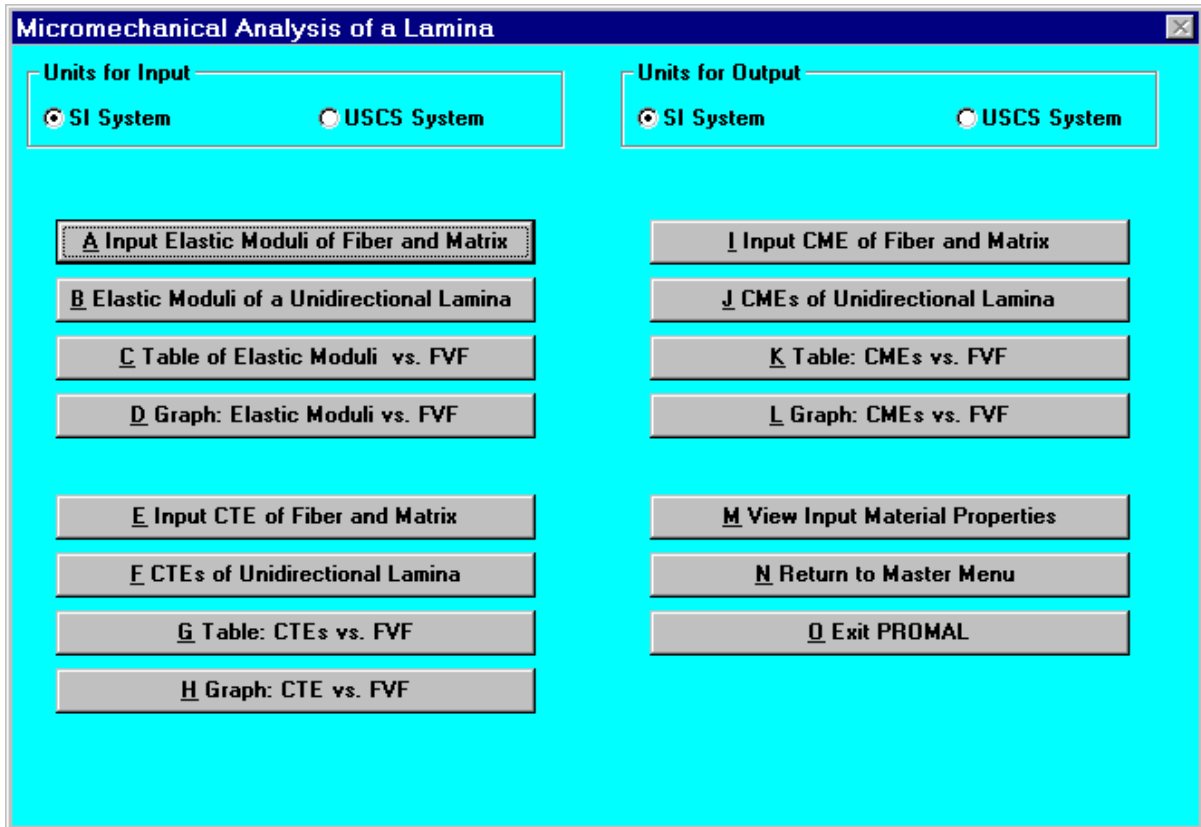
A Quick Example

Task
The following are the mechanical properties of the constituents of a Glass/Epoxy lamina:

- Fiber Young's modulus = 70 GPa
- Matrix Young's modulus = 3.5 GPa
- Fiber Poisson's ratio = 0.2
- Matrix Poisson's ratio = 0.35
- Fiber coefficient of thermal expansion = 0.5E-05 m/m/°C
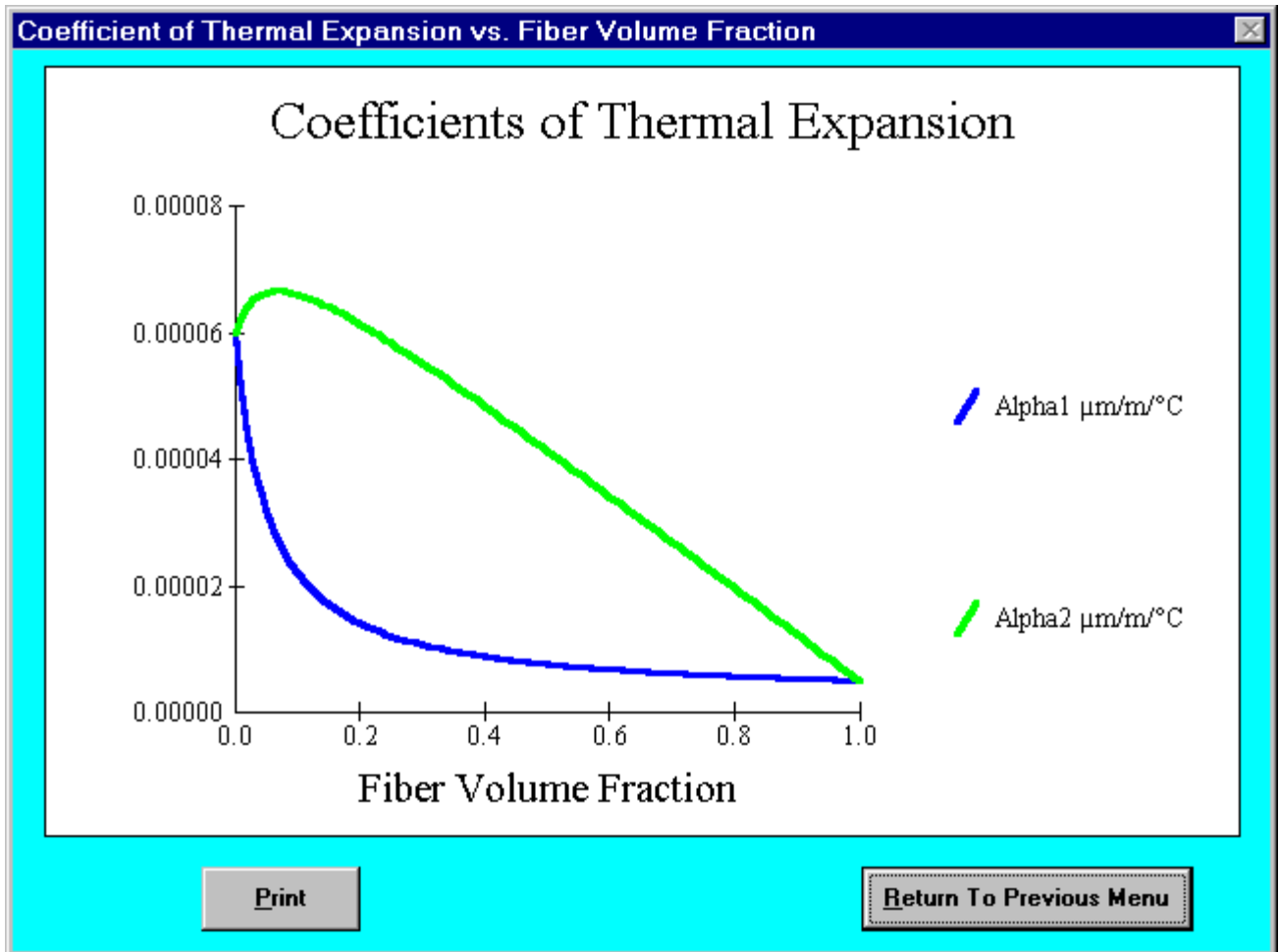- Matrix coefficient of thermal expansion = 6.0E-05 m/m/°C

Find the fiber volume fraction for which the thermal expansion coefficient perpendicular to the fibers is a maximum.

Procedure:
Click on Input Elastic Moduli of Fiber and Matrix Button to enter the elastic moduli of the fiber and the matrix.
Click on Input CTE for Fiber and Matrix Button to enter the coefficient of thermal expansion for fiber and matrix. Note your units!
Click on the Graph: CTEs vs. FVF Button.

Coefficients of Thermal Expansion

— Alpha1 µm/m/°C

— Alpha2 µm/m/°C

Fiber Volume Fraction

Print

Return To Previous Menu

From the graph, as shown in above Figure, we see that the maximum transverse thermal expansion coefficient occurs when the fiber volume fraction is between 0.05 and 0.1. Next, using these values as the upper and lower limits, iteratively use the CTEs of Unidirectional Lamina Button to find the fiber volume fraction for which the thermal expansion coefficient is a maximum. Following this method, a fiber volume fraction of 0.071 is found to produce the maximum transverse coefficient of thermal

expansion.

Coefficient of Thermal Expansion vs. Fiber Volume Fraction
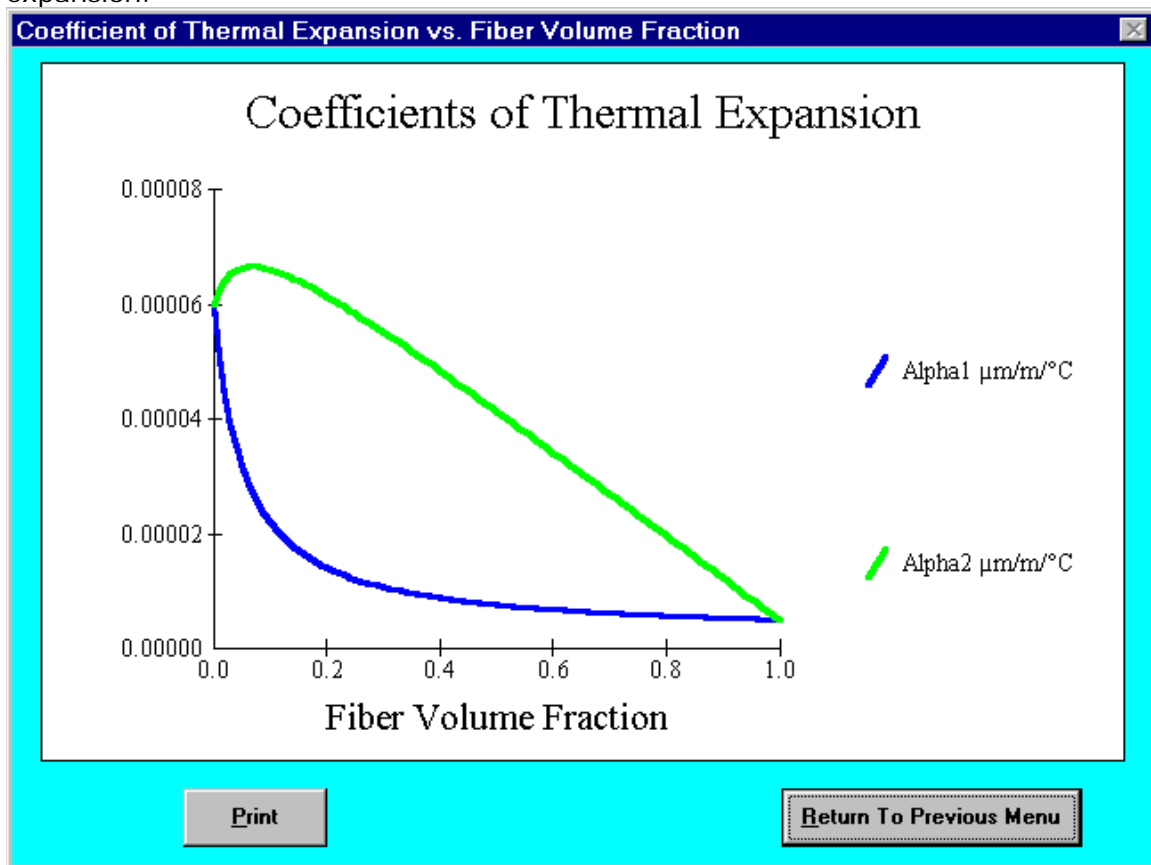
Macromechanical Analysis of a Laminate

The sixth program conducts the macromechanical analysis of a laminate. The laminate can consist of a single type of lamina, hybrids, and sandwich panels. The material properties for the lamina are selected from the material database of the second program. Next you enter the orientation of the plies and the corresponding material number of each ply in the laminate. You can also read laminate stacking sequences from a data file. Now you can display the orientation and ply properties, the stiffness and the compliance matrices, and the equivalent plate properties of the laminate. Enter the mechanical and hygrothermal loads to calculate the global and local stresses and strains, strength ratios at the top, middle and bottom surface of each ply in the laminate.
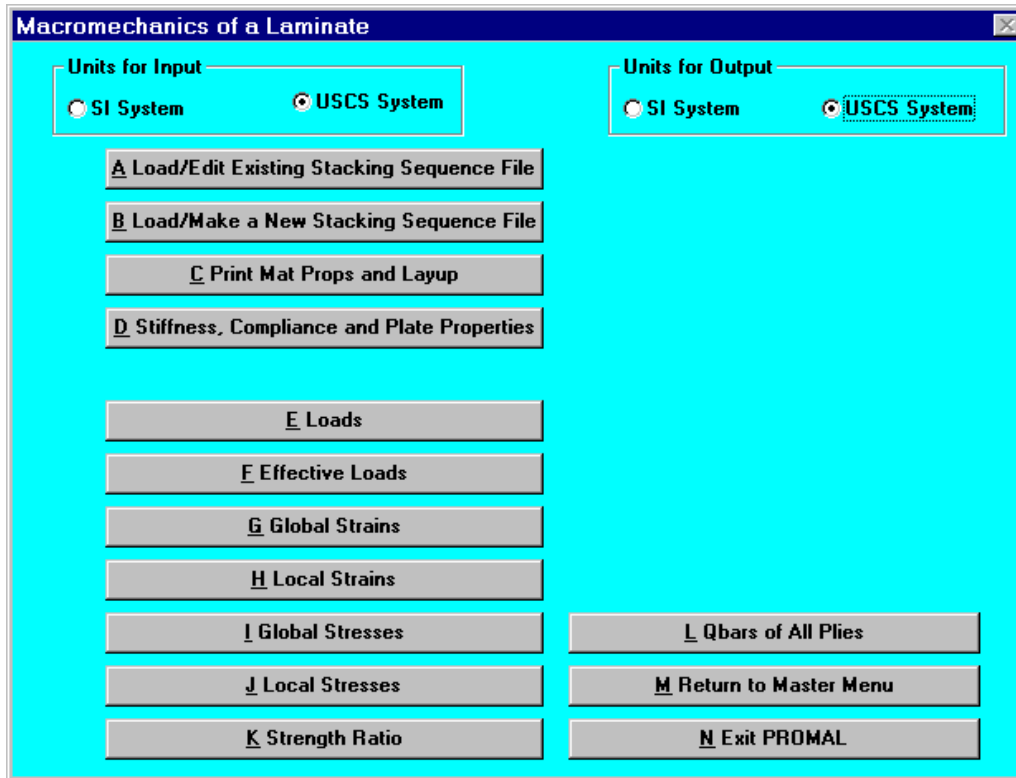
A Quick Example

Task:
Determine the uniaxial stress you can apply in the x-direction to a (0/45/60/90)
Graphite Epoxy laminate. The x-direction is parallel to the fibers in the 0° ply. Use
the maximum stress criterion as a basis for your answer. Use the properties of
unidirectional Graphite/Epoxy from the lamina properties database.

Procedure:
Click on the Macromechanics of a Laminate Button in the Master Menu.
Click on the Load/Make a New Stacking Sequence File Button to make a new
laminate stacking sequence. Since the subscript "s" on the laminate stacking
sequence denotes a symmetric laminate, enter eight for the number of plies. Choose
the options of a single material and a symmetric laminate. Double click on
Graphite/Epoxy in the Material ListBox to flood the material number column in the
laminate stacking sequence grid or enter the material number manually in the grid.
Enter the angle of the top four plies from top to bottom. Click on Save/Return to
Main Menu Button to save the data file.
The task is to find the maximum uniaxial load you can apply in the X-Direction based
on the Maximum Stress failure theory.
Click on Applied Mechanical and Hygrothermal Loads Button to enter a unit force per
unit length in the X-direction, that is, Nx=1 lb/in. Click on Strength Ratios Button to
view the table of strength ratios and are given below.

Laminate Stacking Sequence

Name: Graphite/Epoxy

SI SYSTEM

E1	(GPa) =	1.810E+02
E2	(GPa) =	1.030E+01
Nu12	=	2.800E-01
G12	(GPa) =	7.170E+00
h	(mms) =	1.250E-01
S1TUlt	(MPa) =	1.500E+03
S1CUlt	(MPa) =	1.500E+03
S2TUlt	(MPa) =	4.000E+01
S2CUlt	(MPa) =	2.460E+02
T12Ult	(MPa) =	6.800E+01
Alpha1	(µ/°C)=	2.000E-02
Alpha2	(µ/°C)=	2.250E+01
Beta1	=	0.000E+00
Beta2	=	6.000E-01

BRITISH SYSTEM

E1	(MPsi) =	2.625E+01
E2	(MPsi) =	1.494E+00
Nu12	=	2.800E-01
G12	(MPsi) =	1.040E+00
h	(Inch) =	4.921E-03
S1TUlt	(KPsi) =	2.176E+02
S1CUlt	(KPsi) =	2.176E+02
S2TUlt	(KPsi) =	5.802E+00
S2CUlt	(KPsi) =	3.568E+01
T12Ult	(KPsi) =	9.863E+00
Alpha1	(µ/°F) =	1.111E-02
Alpha2	(µ/°F) =	1.250E+01
Beta1	=	0.000E+00

Material Number	Material Name
1	Graphite/Epoxy
2	Boron/Epoxy
3	Glass/Epoxy
4	Aluminum
5	Steel

The Selected Laminate Stacking Sequence has 8 Plies

The Selected Laminate is Symmetric

Single Click on the Material of Your Choice in the List Box to View Its Properties. Double Click on The Material of Your Choice in The Above List Box To Select the Material of Your Laminate Stacking Sequence.

Ply Num.	Material Num.	Angle of Ply
1	1	0
2	1	45
3	1	60
4	1	90

[Cancel] [Save File/Return to Main Menu]

Ply by Ply Strength Ratios

Strength Ratios

Position	Max Strain	Max Stress	Tsai-Hill	Tsai-Wu	
Top	2.729E+03(1T)	2.720E+03(1T)	2.264E+03	2.166E+03	Ply#1
Middle	2.729E+03(1T)	2.720E+03(1T)	2.264E+03	2.166E+03	Ang=000.000
Bottom	2.729E+03(1T)	2.720E+03(1T)	2.264E+03	2.166E+03	Mat#1
Top	1.698E+03(2T)	1.585E+03(2T)	1.388E+03	1.329E+03	Ply#2
Middle	1.698E+03(2T)	1.585E+03(2T)	1.388E+03	1.329E+03	Ang=045.000
Bottom	1.698E+03(2T)	1.585E+03(2T)	1.388E+03	1.329E+03	Mat#1
Top	1.302E+03(2T)	1.316E+03(2T)	1.270E+03	1.236E+03	Ply#3
Middle	1.302E+03(2T)	1.316E+03(2T)	1.270E+03	1.236E+03	Ang=060.000
Bottom	1.302E+03(2T)	1.316E+03(2T)	1.270E+03	1.236E+03	Mat#1
Top	1.279E+03(2T)	1.298E+03(2T)	1.262E+03	1.229E+03	Ply#4
Middle	1.279E+03(2T)	1.298E+03(2T)	1.262E+03	1.229E+03	Ang=090.000
Bottom	1.279E+03(2T)	1.298E+03(2T)	1.262E+03	1.229E+03	Mat#1

[Print]   [Dump]   [Return to Previous Menu]

Minimum obtained value of strength ratio
$= 1.298 \times 10^3$ (90° ply)
Maximum value of uniaxial load Nx
= Load x Strength Ratio
$= 1.298 \times 10^3$ lb/in.
Thickness of the laminate
= Number of plies x Thickness of one ply
= 0.039368 inches.
Maximum allowable uniaxial stress in X-direction
= Maximum value of load/ Thickness of laminate
$= 1.298 \times 103/0.039368$
= 32.971 Ksi.
Minimum obtained value of strength ratio
$= 1.298 \times 10^3$ (90° ply)
Maximum value of uniaxial load Nx
= Load x Strength Ratio
$= 1.298 \times 10^3$ lb/in.
Thickness of the laminate
= Number of plies x Thickness of one ply
= 0.039368 inches.
Maximum allowable uniaxial stress in X-direction

= Maximum value of load/Thickness of laminate

= $1.298 \times 10^3 / 0.039368$

= 32.971 Ksi.